

# Maths

## Frequency Distribution

We divide data into classes

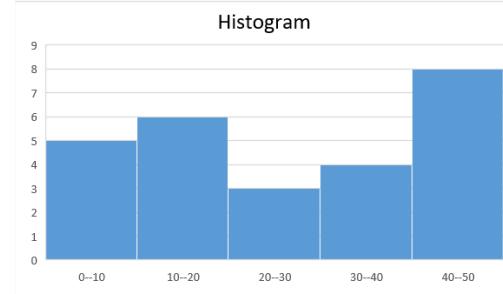
Data: 3, 4, 4, 12, 15, 31, 32, 36, 38, 41, 45, 46, 46, 46, 37, 39

Class(x)	0-10	10-20	20-30	30-40	40-50
Frequency(f)	3	2	1	6	4

Can be turned to :

## Histogram (Bar Chart)

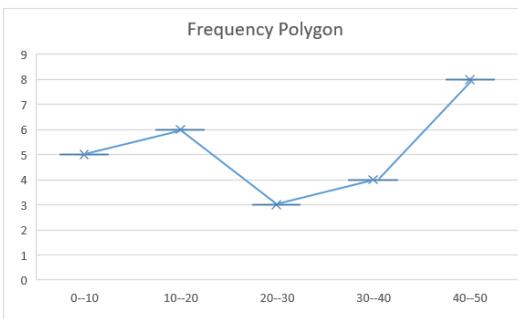
- 1- X-axis represents classes and must be continuous (if a class frequency it's =0 )
- 2- Y-axis represents frequency , we can scale it by dividing by largest frequency.



has no  
the

## Frequency Polygon ( joined lines )

We connect between the midpoints of every bar



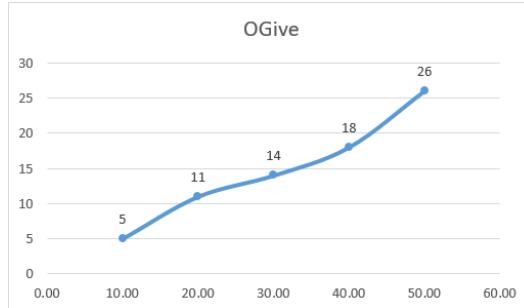
## Comulative frequency polygons ( OGIVE )

We make a new table of  $x_i$  and  $y_i$

$x_i$  : is the largest value in the class border

$y_i$ : add the class's frequency to the frequency of the previous ones

$x_i$	10	20	30	40	50
$y_i$	3	5	6	12	16

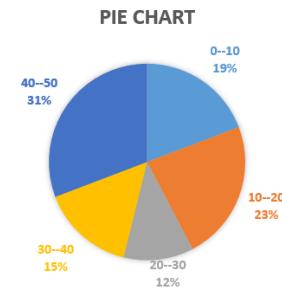


### Circle ( Pie chart )

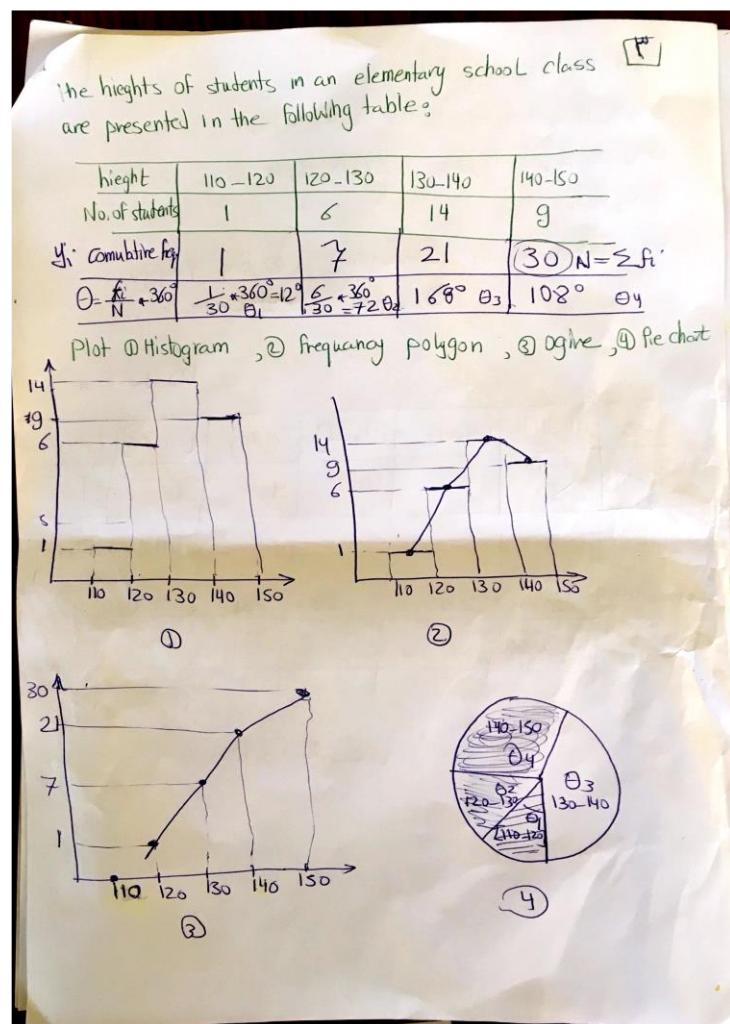
We divide the circle into segments , every segment's angle depends on it's frequency according to

$$\theta_i = \frac{f_i}{N} (2\pi) = \frac{f_i}{N} (360)$$

$$N = \sum f_i$$



Good example on how to solve :



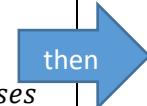
For any data I can get :

(1) Average

Mean  
Median  
Mode

(2) Standard Deviation S.D  $\sigma$

### 1-1 Mean

With repeat	Without repeat	Short-cut method
$\text{Mean}(\mu) = \frac{\sum f_i x_i}{\sum f_i}$ <p><math>x_i</math>: is the midpoint of class period if class is a number then it is <math>x_i</math></p>	$a = \frac{\sum x_i}{n}$ <p><math>a</math> : mean of centres of classes</p>	 $\mu = a + \frac{\sum f_i d_i}{N}$ $; d_i = x_i - a$ $a : \text{any number}$

### 2 Standard Deviation $\sigma$

	Shortest method
$\sigma = \sqrt{\left( \frac{\sum f_i (x_i - \mu)^2}{\sum f_i} \right)}$	$\sigma = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left( \frac{\sum f_i d_i}{\sum f_i} \right)^2}$

### Median

For normal set of numbers

$n$ (odd)	$n$ (even)
$\frac{1}{2}(n + 1)$	$\frac{1}{2}n$ , $\frac{1}{2}(n + 1)$

For a table

We get  $\frac{\sum f_i}{2}$  and search for it in the cumulative column then the class is the median class

After determining the median class (looking at cumulative) then,

$$\text{median} = L + \frac{\frac{\sum f_i}{2} - C}{f} * i$$

$L$  : lower limit of median's class

$f$  : frequency of median class

$C$  : cumulative frequency of previous class

$i$  : width of median class

$N$  :  $\sum f_i$

## Mode

We determine its place in the class with largest frequency

$$Mode = L + \frac{f - f_{-1}}{2f - f_{-1} - f_1} * i$$

$L$  : lower limit of mode class

$f$  : frequency of mode class

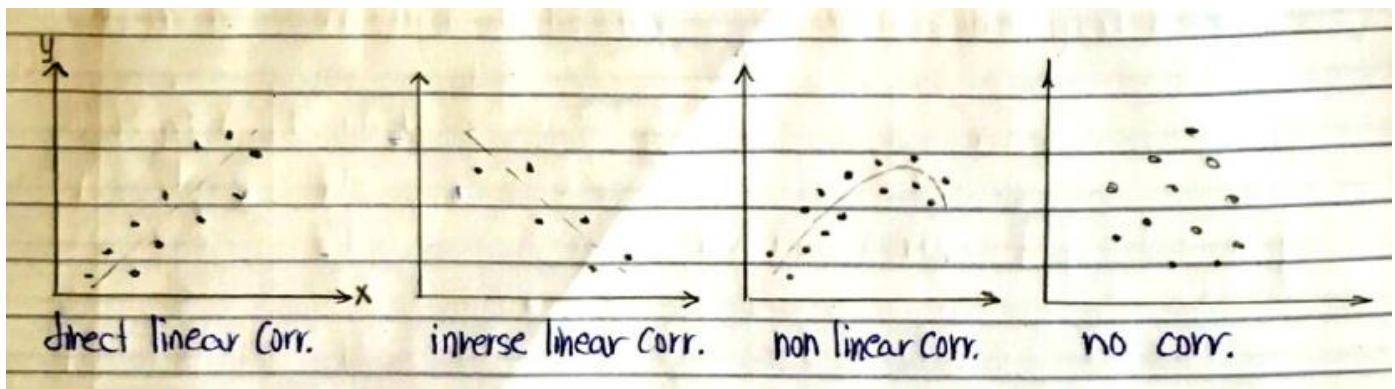
$f_{-1}$  : ~ ~ before

$f_1$  : ~ ~ after

$i$  : width of mode class

## Scatter diagram

To Show points  $(x,y)$  on a rectangular coordinate system



## Correlation coefficient

Quantitative (قيمة عددية)	Quantitative ; Qualitative بيانات وصفية - او قيم تدل على الترتيب
Linear , Pearson "r"	Rank , Spearman "ρ"
$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{n \sum x^2 - (\sum x)^2} \sqrt{n \sum y^2 - (\sum y)^2}}$	$\rho = 1 - \left[ \frac{6 \sum d^2}{n(n^2 - 1)} \right]$ ( $d$ : rank $x$ - rank $y$ )

## Type of correlation

$-1 \leq \text{corr. coeff.} \leq 1$				
1	$\sim 0.5$	0	+ve	-ve
Complete corr	Medium	No corr	direct	inverse
	$> 0.5$ Strong	$< 0.5$ Weak		

## Prediction; Regression

Get prediction equation ( relation between x,y )

Linear predict equation  $y = a + bx$

$$b = \frac{n \sum xy - \sum x \sum y}{n \sum x^2 - (\sum x)^2}$$

$$a = \bar{Y} - b \bar{X}$$

$$\bar{X} = \frac{\sum x}{n} ; \bar{Y} = \frac{\sum y}{n}$$

from that equation we can get value of any y as x known

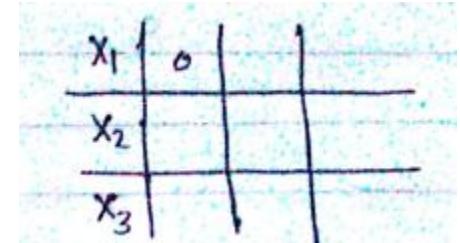
## Multiple Correlation

We get r for every pair

$$r_{12} = \frac{n \sum x_1 x_2 - \sum x_1 \sum x_2}{\sqrt{n \sum x_1^2 - (\sum x_1)^2} \sqrt{n \sum x_2^2 - (\sum x_2)^2}}$$

$$r_{13} = \frac{n \sum x_1 x_3 - \sum x_1 \sum x_3}{\sqrt{n \sum x_1^2 - (\sum x_1)^2} \sqrt{n \sum x_3^2 - (\sum x_3)^2}}$$

$$r_{23} = \frac{n \sum x_2 x_3 - \sum x_2 \sum x_3}{\sqrt{n \sum x_2^2 - (\sum x_2)^2} \sqrt{n \sum x_3^2 - (\sum x_3)^2}}$$



$$x_1 = \beta_1 + \beta_2 x_2 + \beta_3 x_3$$

To get  $\beta_1, \beta_2, \beta_3$  we have 2 methods

### method (1)

$$x_1 = \beta_1 + \beta_2 x_2 + \beta_3 x_3$$

$$* \sum \quad \sum x_1 = n\beta_1 + \beta_2 \sum x_2 + \beta_3 \sum x_3 \quad (1)$$

$$* x_2 \rightarrow * \sum \quad \sum x_1 x_2 = \beta_1 \sum x_2 + \beta_2 \sum x_2^2 + \beta_3 \sum x_2 x_3 \quad (2)$$

$$* x_3 \rightarrow * \sum \quad \sum x_1 x_3 = \beta_1 \sum x_3 + \beta_2 \sum x_2 x_3 + \beta_3 \sum x_3^2 \quad (3)$$

We solve 1,2,3 together to get  $\beta_1, \beta_2, \beta_3$

### method (2)

$$\sigma_1 = \sqrt{\frac{\sum x_1^2}{n} - \left(\frac{\sum x_1}{n}\right)^2}$$

$$\sigma_2 = \sqrt{\frac{\sum x_2^2}{n} - \left(\frac{\sum x_2}{n}\right)^2}$$

$$\sigma_3 = \sqrt{\frac{\sum x_3^2}{n} - \left(\frac{\sum x_3}{n}\right)^2}$$

$$\text{Note : } \frac{\sum x_1}{n} = \bar{X}_1 = \mu_{x_1}$$

$$x_1 = \beta_2 x_2 + \beta_3 x_3$$

$$\beta_2 = \frac{\sigma_1}{\sigma_2} \left( \frac{r_{12} - r_{13}r_{23}}{1 - r_{23}^2} \right)$$

$$\beta_3 = \frac{\sigma_1}{\sigma_3} \left( \frac{r_{13} - r_{12}r_{23}}{1 - r_{23}^2} \right)$$

## Proofs

Show that for any value of  $a$  if  $d_i = x_i - a$  that  $\mu = a + \frac{\sum f_i d_i}{\sum f_i}$

$$\mu = \frac{\sum x_i f_i}{\sum f_i} \quad \text{but } d_i = x_i - a \rightarrow x_i = d_i + a$$

$$\mu = \frac{\sum (d_i + a) f_i}{\sum f_i} = \frac{\sum f_i d_i}{\sum f_i} + \frac{\sum f_i a}{\sum f_i} = \frac{\sum f_i d_i}{\sum f_i} + a \frac{\sum f_i}{\sum f_i} = a + \frac{\sum f_i d_i}{\sum f_i}$$

$$\text{shortest method , prove } \sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$$

$$\sigma = \sqrt{\frac{\sum f_i (x_i - \mu)^2}{\sum f_i}} \quad \text{put : } \mu = \frac{\sum x_i f_i}{\sum f_i}$$

$$\sigma^2 = \frac{1}{\sum f_i} \sum f_i [x_i^2 - 2\mu x_i + \mu^2] = \frac{1}{\sum f_i} \left[ \sum f_i x_i^2 - \frac{2(\sum f_i x_i)^2}{\sum f_i} + \frac{(\sum f_i x_i)^2 \sum f_i}{(\sum f_i)^2} \right]$$

$$\sigma^2 = \frac{1}{\sum f_i} \left[ \sum f_i x_i^2 - \frac{\sum f_i x_i^2}{\sum f_i} \right]$$

$$\sigma = \sqrt{\frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2}$$

show that for any value a  $\sigma = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$

$$\begin{aligned} d_i &= x_i - a \\ \sum f_i &= N \end{aligned}$$

$$\begin{aligned} \sigma &= \frac{1}{N} \sum f_i [(x_i - a) - (\mu - a)]^2 = \frac{1}{N} \sum f_i (d_i - (\mu - a))^2 \\ &= \frac{1}{N} \left[ \sum f_i d_i^2 - 2(\mu - a) \sum f_i d_i + (\mu - a)^2 \frac{\sum f_i}{N} \right] \end{aligned}$$

$$\begin{aligned} \mu &= a + \frac{\sum f_i d_i}{N} \\ \sigma &= \frac{1}{N} \left[ \sum f_i d_i^2 - 2 \frac{(\sum f_i d_i)^2}{N} + \frac{(\sum f_i d_i)^2}{N} \right] \end{aligned}$$

$$\sigma = \sqrt{\frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$$

let  $\mu_x$  and  $\mu_y$  be means of  $x_i$  and  $y_i$ ,  $X = x - \mu_x, Y = y - \mu_y$  show that

$$r = \frac{\sum XY}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})} = \frac{\sum XY}{n \sigma_x \sigma_y} = \frac{\text{cov}(x, y)}{\sigma_x \sigma_y}$$

where  $\sigma_x, \sigma_y$  are standard deviation of  $x_i$  and  $y_i$

$$f_i = 1, \sum f_i = n$$

$$\sigma_x^2 = \frac{\sum f_i (x - \mu_x)^2}{\sum f_i} = \frac{1}{n} \sum (x - \mu_x)^2$$

$$\sigma_x^2 = \frac{1}{n} \sum X^2 \quad \sigma_y^2 = \frac{1}{n} \sum Y^2$$

$$\sum XY = \sum (x - \mu_x)(y - \mu_y) = \sum xy - \mu_y \sum x - \mu_x \sum y + \mu_x \mu_y \sum 1 =$$

$$\mu_x = \frac{\sum f_i x_i}{\sum f_i} = \frac{1}{n} \sum x \quad ; \mu_y = \frac{1}{n} \sum y$$

$$\sum XY = \sum xy - \frac{1}{n} \sum x \sum y + \frac{1}{n} \sum x \sum y$$

$$\therefore \sum XY = \sum xy - \frac{1}{n} \sum x \sum y \rightarrow (1)$$

$$\sum X^2 = \sum (x - \mu_x)^2 = \sum x^2 - 2\mu_x \sum x + \mu_x^2 \sum 1$$

$$\sum X^2 = \sum x^2 - 2n\mu_x^2 + n\mu_x^2 = \sum x^2 - n\mu_x^2 = \sum x^2 - n \left( \frac{\sum x}{n} \right)^2 = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$\therefore \sqrt{\sum X^2} = \sqrt{\frac{1}{n} [n \sum x^2 - (\sum x)^2]}$$

$$\therefore \sqrt{\sum Y^2} = \sqrt{\frac{1}{n} [n \sum y^2 - (\sum y)^2]}$$

$$\text{then : } r = \frac{\sum XY}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})} = \frac{\sum xy - \frac{1}{n} \sum x \sum y}{\left( \sqrt{\frac{1}{n} [n \sum x^2 - (\sum x)^2]} \right) \left( \sqrt{\frac{1}{n} [n \sum y^2 - (\sum y)^2]} \right)}$$

$$\frac{\sum xy - \frac{1}{n} \sum x \sum y}{\frac{1}{n} \left( \sqrt{[n \sum x^2 - (\sum x)^2]} \right) \left( \sqrt{[n \sum y^2 - (\sum y)^2]} \right)} = \frac{n \sum xy - \sum x \sum y}{\left( \sqrt{[n \sum x^2 - (\sum x)^2]} \right) \left( \sqrt{[n \sum y^2 - (\sum y)^2]} \right)} \quad \#1$$

$$\sigma_x^2 = \frac{1}{n} \sum X^2 \quad ; \quad \sigma_y^2 = \frac{1}{n} \sum Y^2$$

$$r = \frac{\sum XY}{\sqrt{n} \sigma_x \sqrt{n} \sigma_y} = \frac{\sum XY}{n \sigma_x \sigma_y} = \frac{\text{cov}(x, y)}{n \sigma_x \sigma_y} \quad \#2, 3$$

show that  $\sigma_{x-y}^2 = \sigma_x^2 + \sigma_y^2 - 2r\sigma_x\sigma_y$

$$\sigma_x = \sqrt{\frac{\sum f(x - \mu_x)^2}{\sum f}} ; \sigma_y = \sqrt{\frac{\sum f(y - \mu_y)^2}{\sum f}}$$

$$\sigma_x^2 = \frac{1}{n} \sum (x - \mu_x)^2 = \frac{1}{n} [\sum x^2 - 2\mu_x \sum x + n\mu_x^2]$$

$$\sigma_y^2 = \frac{1}{n} [\sum y^2 - 2\mu_y \sum y + n\mu_y^2]$$

$$r = \frac{\sum (x - \mu_x)(y - \mu_y)}{n \sigma_x \sigma_y}$$

$$\text{L.H.S} = \sigma_{x-y}^2 = \frac{1}{n} \sum [(x - y) - \mu_{x-y}]^2$$

$$\mu_{x-y} = \frac{\sum (x - y)}{n} = \frac{\sum x}{n} - \frac{\sum y}{n}$$

$$\mu_{x-y} = \mu_x - \mu_y$$

$$\sigma_{x-y}^2 = \frac{1}{n} \sum [(x - \mu_x) - (y - \mu_y)]^2 = \frac{1}{n} \sum [(x - \mu_x) - (y - \mu_y)]^2$$

$$= \frac{1}{n} \sum [(x - \mu_x)^2 - 2(x - \mu_x)(y - \mu_y) + (y - \mu_y)^2]$$

$$= \sigma_x^2 - 2r\sigma_x\sigma_y + \sigma_y^2 \quad \#$$

let  $x_i, y_i$  be ranked individual distinct data then  $r = 1 - \frac{6 \sum d^2}{n(n^2 - 1)}$

$$\text{use : } d_i = x_i - y_i, \sum_{i=1}^n i^2 = \frac{n(n+1)(2n+1)}{6}; X = x - \mu_x, Y = y - \mu_y; r = \frac{\sum XY}{(\sqrt{\sum X^2})(\sqrt{\sum Y^2})}$$

$$\text{The mean of Rank x} = \frac{1 + 2 + 3 + \dots + n}{n}$$

$$\text{Sum of arithmetic series} = \frac{n}{2} (a_1 + a_n)$$

so,

$$\mu_{R_x} = \frac{1}{n} \left[ \frac{n}{2} (1+n) \right] = \frac{n+1}{2}$$

$$\mu_{R_y} = \frac{n+1}{2}$$

$$d_i = X_i - Y_i = (x - \mu_{R_1}) - (y - \mu_{R_2}) = x_i - y_i$$

بدلاًه الترتيب  $\Sigma x^2$  ايجاد كل جزء في الـ

$$\Sigma X^2 = \Sigma (x - \mu_{R_1})^2 = \sum_{i=1}^n \left[ x_i - \frac{n+1}{2} \right]^2$$

$$\Sigma X^2 = \Sigma x^2 - (n+1)\Sigma x + \left( \frac{n+1}{2} \right)^2 \Sigma 1 \rightarrow (1)$$

$$\Sigma x^2 = 1^2 + 2^2 + \dots + n^2 = \frac{(n(n+1)(2n+1))}{6}$$

$$\text{same, } \Sigma y^2 = \frac{(n(n+1)(2n+1))}{6}$$

$$\Sigma x = 1 + 2 + \dots + n = \frac{n}{2}[1 + n]$$

$$\text{same, } \Sigma y = 1 + 2 + \dots + n = \frac{n}{2}[1 + n]$$

subs in (1)

$$\begin{aligned} \Sigma X^2 &= \Sigma x^2 - (n+1)\Sigma x + \left( \frac{n+1}{2} \right)^2 \Sigma 1 \\ &= \frac{(n(n+1)(2n+1))}{6} - \frac{n}{2}(n+1)^2 + \left( \frac{n+1}{2} \right)^2 n \\ &= \frac{n(n+1)}{2} \left[ \frac{2n+1}{3} - n - 1 + \frac{n+1}{2} \right] = \frac{n(n+1)}{2} \left[ \frac{4n+2-6n-6+3n+3}{6} \right] \\ &= \frac{n(n+1)}{2} \left[ \frac{n-1}{6} \right] = \frac{n(n^2-1)}{12} \\ \Sigma X^2 &= \frac{n(n^2-1)}{12} \\ \Sigma Y^2 &= \frac{n(n^2-1)}{12} \end{aligned}$$

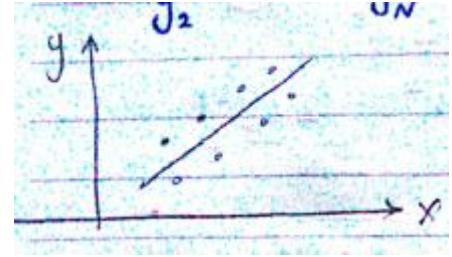
$$\begin{aligned} \Sigma XY &= \Sigma (x - \mu_{R_1})(y - \mu_{R_2}) \\ \Sigma d^2 &= \Sigma (X - Y)^2 = \Sigma X^2 - 2\Sigma XY + \Sigma Y^2 \\ \Sigma XY &= \frac{1}{2} [\Sigma X^2 + \Sigma Y^2 - \Sigma d^2] = \frac{1}{2} \left[ \frac{n(n^2-1)}{12} - \Sigma d^2 \right] \end{aligned}$$

subs in r

$$r = \frac{\Sigma XY}{(\sqrt{\Sigma X^2})(\sqrt{\Sigma Y^2})} = \frac{\frac{1}{2} \left[ \frac{n(n^2-1)}{12} - \Sigma d^2 \right]}{\frac{n(n^2-1)}{12}} = 1 - \frac{6\Sigma d^2}{n(n^2-1)} \#$$

Get the value of  $a, b$  in regression line  $y = a + bx$

$$\begin{aligned}y_1 &= a + bx_1 \\y_2 &= a + bx_2 \\&\vdots \\y_n &= a + bx_n\end{aligned}$$



$$y_1 + y_2 + \dots + y_n = \sum y = na + b \sum x_i \rightarrow (*)$$

$$\therefore \sum x_i y_i = a \sum x_i + b \sum x_i^2 \rightarrow (**) \quad \text{dividing * by } N$$

$$\mu_y = a + b \mu_x$$

$$a = \frac{\sum y_i}{N} - b \frac{\sum x_i}{N}$$

$$\sum x_i y_i = \frac{\sum x_i \sum y_i}{N} - b \frac{(\sum x_i)^2}{N} + b \sum x_i^2$$

$$\sum x_i y_i - \frac{(\sum x_i)(\sum y_i)}{N} = b \left[ \sum x_i^2 - \frac{(\sum x_i)^2}{N} \right]$$

$$\therefore a = \mu_y - b \mu_x$$

$$b = \frac{N \sum x_i y_i - (\sum x_i)(\sum y_i)}{N [\sum x_i^2] - (\sum x_i)^2}$$

show that the slope of the regression line

$$y = a + bx, \text{ is } r \frac{\sigma_y}{\sigma_x}$$

$$X = x - \mu_x; Y = y - \mu_y$$

$$r = \frac{\sum XY}{\sqrt{\sum X^2} \sqrt{(\sum Y^2)}}; \sigma_x = \sqrt{\frac{\sum (x - \mu_x)^2}{n}} = \sqrt{\frac{\sum X^2}{n}}; \sigma_y = \sqrt{\frac{\sum Y^2}{n}}$$

$$y = a + bx \rightarrow \sum y = na + b \sum x$$

$$\sum xy = a \sum x + b \sum x^2$$

$$\mu_y = a + b \mu_x$$

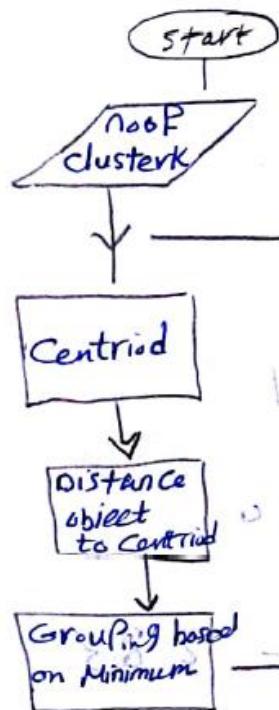
$$\sum XY = \sum (x - \mu_x)(y - \mu_y) = \sum (x - \mu_x)(a + bx - \mu_y) = \sum (x - \mu_x)(a + bx - a - b \mu_x)$$

$$b \sum (x - \mu_x)^2 = b \sum X^2$$

$$r = \frac{\sum XY}{\sqrt{\sum X^2} \sqrt{\sum Y^2}} = \frac{b \sum X^2}{\sqrt{\sum X^2} \sqrt{\sum Y^2}} = \frac{b \left( \sqrt{\sum X^2} \right)}{\left( \sqrt{\sum Y^2} \right)} * \frac{1}{\sqrt{n}} = \frac{b \left( \frac{\sqrt{\sum X^2}}{\sqrt{n}} \right)}{\left( \frac{\sqrt{\sum Y^2}}{\sqrt{n}} \right)} = \frac{b \sigma_x}{\sigma_y}$$

$$\therefore b = \text{slope} = \frac{r \sigma_y}{\sigma_x}$$

## K - mean clustering



1- تكرر عدد التجمعات  
 2- لفترة مراقبة التجمعات  
 3- تنتهي بعد حل معنير صدر المراقب  
 4- تقسّم الـ data مبنية على المراقب  
 5- يختبر المراقب للتقسيم الجيد فإذا كانت  
 صفات القسمة تتفق واداءً تغير تكرر

$$d(C_1 A) = \sqrt{(-)^2 + (-)^2} ; d(C_2 A) = \sqrt{(-)^2 + (-)^2}$$

$$d(C_1 B) = \sqrt{(-)^2 + (-)^2} ; d(C_2 B) = \sqrt{(-)^2 + (-)^2}$$

$$d(C_1 C) = \sqrt{(-)^2 + (-)^2} ; d(C_2 C) = \sqrt{(-)^2 + (-)^2}$$

$$D_0 = \begin{bmatrix} s & & \\ & s & \\ & & s \end{bmatrix} \therefore G_1 = \{A\} ; G_2 = \{B, C\}$$

$$\text{new centroids } C_{11} (x_A, y_A) ; C_{12} = \left( \frac{x_B + x_C}{2}, \frac{y_B + y_C}{2} \right)$$

REPEAT

## Probabilities

- 1- Random Experiment : التجربة الغير معروف نتائجها
- 2- Sample Space (S) : هو الفضاء الذي يحتوي جميع نواتج التجربة العشوائية
- 3- Event : هو حدث يحدث داخل التجربة العشوائية
- 4- Probability of Event  $P(A) = \frac{\text{no of A elements}}{\text{no of S elements}}$

## الفروض الاحتمالية Probability axioms

- 1 -  $0 \leq P(A) \leq 1$
- 2 -  $P(S) = 1$

3 – let  $A, B$  are mutually exclusive even  $A \cap B = \phi$   
 $\therefore P(A \cup B) = P(A) + P(B)$

### Basic Probability Theory

- 1)  $P(\phi) = 0$
- 2)  $P(A^c) = 1 - P(A)$
- 3)  $A \subseteq B \rightarrow P(A) \leq P(B)$
- 4)  $P(A - B) = P(A) - P(A \cap B)$
- 5)  $P(A \cup B) = P(A + B) = P(A) + P(B) - P(A \cap B)$

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### *Proofs*

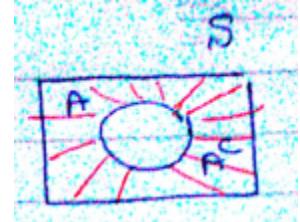
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1)  $P(\phi) = 0$

$$\begin{aligned} A &= A \cup \phi \\ P(A) &= P(A \cup \phi) \\ P(A) &= P(A) + P(\phi) \text{ then } P(\phi) = 0 \end{aligned}$$

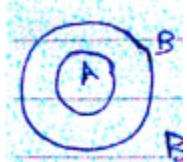
2)  $P(A^c) = 1 - P(A)$

$$\begin{aligned} S &= A \cup A^c \\ A^c \cap A &= \phi \\ P(S) &= P(A \cup A^c) \\ 1 &= P(A) + P(A^c) \\ \therefore P(A^c) &= 1 - P(A) \end{aligned}$$



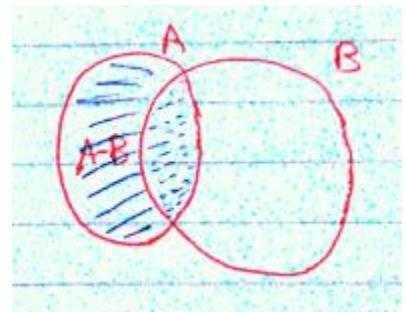
3)  $A \subseteq B \rightarrow P(A) \leq P(B)$

$$\begin{aligned} P(A) &\leq P(B) \\ B &= (B - A) \cup A \\ (B - A) \cap A &= \phi \\ P(B) &= P((B - A) \cup A) \\ &\stackrel{+ve > 0}{=} P(B - A) + P(A) \\ \therefore P(B) &\geq P(A) \end{aligned}$$



4)  $P(A - B) = P(A) - P(A \cap B)$

$$\begin{aligned} A &= (A - B) \cup (A \cap B) \\ (A - B) \cap (A \cap B) &= \phi \\ P(A) &= P(A - B) + P(A \cap B) \\ P(A - B) &= P(A) - P(A \cap B) \end{aligned}$$



$$5) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

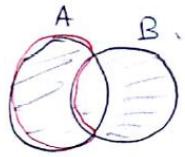
$$A \cup B = (A - B) \cup B$$

$$(A - B) \cap B = \phi$$

$$P(A \cup B) = P(A - B) + P(B)$$

$$= P(A) - P(A \cap B) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



6) if  $A, B$  are independent events then

1 -  $A^c, B$  are independent

2 -  $A, B^c$  are independent

3 -  $A^c, B^c$  are independent

independency  $P(A \cap B) = P(A)P(B)$

$$(1) \quad P(A^c \cap B) = P(A^c)P(B)$$

$$B - A = A^c \cap B$$

$$P(B - A) = P(B) - P(A \cap B)$$

$$P(A^c \cap B) = P(B) - P(A)P(B)$$

$$= P(B)(1 - P(A)) = P(B)P(A^c)$$

$\therefore A^c, B$  are independent

$$(2) \quad P(A \cap B^c) = P(A)P(B^c)$$

$$A \cap B^c = A - B$$

$$P(A - B) = P(A) - P(A \cap B)$$

$$P(A \cap B^c) = P(A) - P(A)P(B)$$

$$= P(A)[1 - P(B)] = P(A)P(B^c)$$

$A, B^c$  independent

$$(3) P(A^c \cap B^c) = P(A^c)P(B^c)$$

$$(A \cup B)^c = A^c \cap B^c$$

$$P(A^c \cap B^c) = P[(A \cup B)^c] = 1 - P(A \cup B)$$

$$= 1 - P(A) - P(B) + P(A \cap B)$$

$$= (1 - P(A)) - P(B) + P(A)P(B)$$

$$= (1 - P(A)) - P(B)(1 - P(A))$$

$$= (1 - P(A))(1 - P(B)) = P(A^c)P(B^c)$$

$A^c, B^c$  independent

$A \text{ or } B (A + B)$	$A \cup B = P(A) + P(B) - P(A \cap B)$
$A \text{ and } B (A \times B)$	$A \cap B = P(A) * P(B)$
$B \text{ only}$	$B \cap A^c \cap C^c$
$A \text{ depended on } B \text{ and conditional}$	$P(A B) = \frac{P(A \cap B)}{P(B)}$
$A \text{ independent on } B \text{ and conditional}$ $\therefore A, B, A^c B^c \text{ are all independent}$	$P(A B) = P(A)$ $P(B A) = P(B)$ $P(A \cap B) = P(A)P(B)$
$A \cap B^c$	$A - B$
$(A \cup B)^c$	$A^c \cap B^c$
$P(A^c \cap B^c)$	$1 - P(A \cup B)$

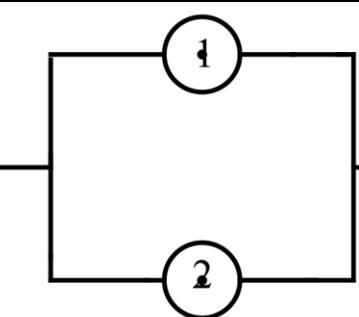
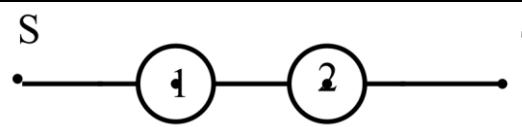
Methods to set probability space

- 1- Tree
- 2- Crossed Lines
- 3- (Possible Events) $^{\wedge}$ (Repetitions)

### CaeSar Cipher

$$P = (C - K) \bmod 26$$

$$C = (P + K) \bmod 26$$

Parallel					Series																																																						
																																																											
$P = 1 + 2$					$P = 1 * 2$																																																						
<table border="1"> <thead> <tr> <th>E</th><th>1</th><th>2</th><th>1+2</th><th></th></tr> </thead> <tbody> <tr> <td><math>E_1</math></td><td>0</td><td>0</td><td>0</td><td></td></tr> <tr> <td><math>E_2</math></td><td>0</td><td>1</td><td>1</td><td></td></tr> <tr> <td><math>E_3</math></td><td>1</td><td>0</td><td>1</td><td></td></tr> <tr> <td><math>E_4</math></td><td>1</td><td>1</td><td>1</td><td></td></tr> </tbody> </table>					E	1	2	1+2		$E_1$	0	0	0		$E_2$	0	1	1		$E_3$	1	0	1		$E_4$	1	1	1		<table border="1"> <thead> <tr> <th>E</th><th>1</th><th>2</th><th><math>1*2</math></th><th></th></tr> </thead> <tbody> <tr> <td><math>E_1</math></td><td>0</td><td>0</td><td>0</td><td></td></tr> <tr> <td><math>E_2</math></td><td>0</td><td>1</td><td>0</td><td></td></tr> <tr> <td><math>E_3</math></td><td>1</td><td>0</td><td>0</td><td></td></tr> <tr> <td><math>E_4</math></td><td>1</td><td>1</td><td>1</td><td></td></tr> </tbody> </table>					E	1	2	$1*2$		$E_1$	0	0	0		$E_2$	0	1	0		$E_3$	1	0	0		$E_4$	1	1	1	
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$S = \{E_1, E_2, E_3, E_4\}$					$S = \{E_1, E_2, E_3, E_4\}$																																																						
$P(A) (s \text{ to connect to } T)$ $P(A) = P(0,1) + P(1,0) + P(1,1)$					$P(A) = P(1,1)$ $P(A) = 1 * (0.8)^2$																																																						
عدد الوحدات في اليفنت الواحد يتبع اس لاحتمالاته الفتح و عدد الاصفار يتبع اس لاحتمالاته عدم الفتح عدد مرات تكرار نفس عدد الوحدات يبقى اضرب $P(A) = 2 * (0.8)^1 (0.2)^1 + 1 * (0.8)^2$																																																											

### R-step Experiment

If R is experiment has r-step to process

1<sup>st</sup> step has n<sub>1</sub> ways to process

2<sup>nd</sup> has n<sub>2</sub> ways

rth step has n<sub>r</sub> ways to process

The total number of ways to process R = n<sub>1</sub>n<sub>2</sub>n<sub>3</sub> ... n<sub>r</sub>

مثال سريع عاوز اعمل من 3 الوان علم افقي او راسي من 4 سترايب بس ميكتش نفس الون في اتنين جنب بعض  
 الحل

$$P(A) = (3)(2)(2)(2) + (3)(2)(2)(2)$$

پیشگیری بخوبی

فلکر مسکن (ات)

If the red colour not appear

JSI JIAD

AJIN JIAD

$$P(A \cap B) = \frac{P(A \cap B)}{P(B)}.$$

## Permutation and Combinations

Combination:

no of ways to choose  $r$  from  $n$  without caring about order

$$C_r^n = \frac{n!}{r!(n-r)!}$$

Permutation :

no of ways to choose  $r$  from  $n$  with order

$$P_r^n = \frac{n!}{(n-r)!}$$

Ex : what's the minimum no of letters to make up 720 words

$$P_3^x$$

$$\text{solution : } \frac{\sqrt{720}}{3} = 9$$

$$9 * 10 * 11 \neq 720$$

$$7 * 8 * 9 \neq 720$$

$$10 * 9 * 8 = 720 \quad \therefore n = 10 \rightarrow P_3^{10} = 720 \rightarrow 10 \text{ letters}$$

$$\text{to be from letter } (A, B, C, D) \rightarrow P = \frac{P_3^4}{P_3^{10}} = \frac{1}{30}$$

## Number of ways in grouping data in classes

### After grouping data in classes

Number of ways of partition  $n$  distinct objects into groups containing  $n_1, n_2, \dots, n_k$

$$= \frac{n!}{n_1! n_2! \dots n_k!}$$

where  $n = n_1 + n_2 + \dots + n_k$

### Proof

Show that the number of ways of partitioning  $n$  objects into groups containing  $n_1, n_2, \dots, n_k$  objects is  $\frac{n!}{n_1! n_2! \dots n_k!}$

We can partition  $n$  objects into groups by  $k$  steps

- First step : shows  $n_1$  from  $n$  to put it in  $G_1 \rightarrow C_{n_1}^n$
- Second step : shows  $n_2$  from  $(n - n_1)$  to put it in  $G_1 \rightarrow C_{n_2}^{n-n_1}$
- .
- .
- .
- $K$  step : shows  $n_k$  from  $n - n_1 - n_2 - \dots - n_{k-1}$  to put it in  $G_1 \rightarrow C_{n_k}^{n-n_1-n_2-\dots-n_{k-1}}$
- Then, total number =  $C_{n_1}^n * C_{n_2}^{n-n_1} * \dots * C_{n_k}^{n-n_1-n_2-\dots-n_{k-1}}$
- $\frac{n!}{n!(n-n_1)!} * \frac{(n-n_1)!}{n!(n-n_1-n_2)!} * \dots * \frac{(n-n_1-n_2-\dots-n_{k-1})!}{n_k!(n-n_1-n_2-\dots-n_{k-1})!} = \frac{n!}{n_1! n_2! \dots n_k!}$

## Total Probability

If sample space  $S$  partitions by  $A_1, A_2, \dots, A_n$  where  $S = A_1 \cup A_2 \dots \cup A_n$  and  $A_i \cap A_j = \emptyset$  and  $B \subset S$

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \dots + P(B|A_n)P(A_n)$$

### Proofs

Show that if  $A_1, A_2, \dots, A_n$  partitions of  $S$  and  $B \subset S \therefore P(B) = \sum_{i=1}^n P(B|A_i)P(A_i)$

$$\therefore B = (B \cap A_1) \cup (B \cap A_2) \cup \dots \cup (B \cap A_n)$$

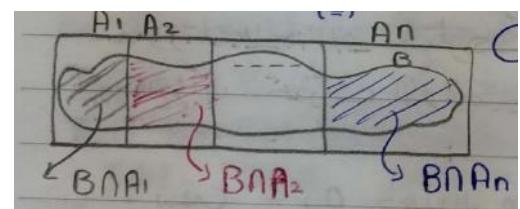
$$\therefore A_i \cap A_j = \emptyset$$

$$\therefore P(B) = P(B \cap A_1) + P(B \cap A_2) + \dots + P(B \cap A_n) = \sum_{i=1}^n P(B \cap A_i)$$

$$\therefore P(B|A_i) = \frac{P(B \cap A_i)}{P(A_i)}$$

$$\therefore P(B|A_i)P(A_i) = P(B \cap A_i)$$

$$\therefore P(B) = \sum_{i=1}^n P(B|A_i)P(A_i)$$



Show that  $P(A_k|B) = \frac{P(B|A_k)P(A_k)}{(P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + \dots + P(B|A_n)P(A_n))}$  where  $S = A_i \cup A_j$ ,  $A_i \cap A_j = \emptyset$

From total probability  $P(B) = \sum_{i=1}^n P(B|A_i)P(A_i)$

$$P(A_k|B) = \frac{P(B \cap A_k)}{P(B)}, P(B|A_k) = \frac{P(B \cap A_k)}{P(A_k)}$$

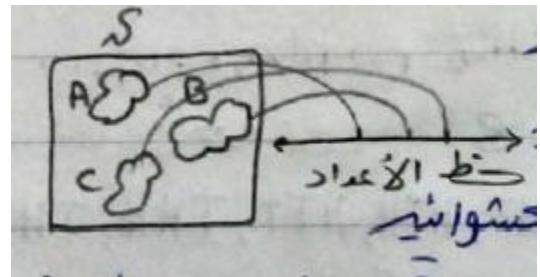
$$\therefore P(B \cap A_k) = P(A_k)P(B|A_k)$$

$$\therefore P(A_k|B) = \frac{P(B \cap A_k)}{P(B)} = \frac{P(A_k)P(B|A_k)}{\sum_{i=1}^n P(B|A_i)P(A_i)} \text{ Bay's Theorem}$$

## Random Variables

We move the problem from words or events to numbers

- 1- Can be discrete (countable)
- 2- Continuous



### Types of Random Variable

Discrete RV	Continuous RV
Finite number (can be listed in a table)	Much more we used periods $a \leq x \leq b$

Note :

$f(x) \rightarrow$  Probability fun

$F(x) \rightarrow$  C.D.F cumulative

### (1) Probability function $f(x)$

Probability Density function P.D.F	Probability Mass Function P.M.F						
<p>Continuous Random variables (C.R.V)</p> <p>Probability density function</p> <p><u>CONDITIONS</u></p> <ol style="list-style-type: none"> <li>(1) <math>f(x_i) \geq 0</math></li> <li>(2) <math>\int_{-\infty}^{\infty} f(x)dx = 1</math></li> </ol> <p>Periods</p> $F(x) = \begin{cases} 0 & -\infty < x < 0 \\ \frac{1}{8} & 0 \leq x < 1 \\ \frac{3}{8} + \frac{1}{8} = \frac{4}{8} & 1 \leq x < 2 \\ \frac{4}{8} + \frac{3}{8} = \frac{7}{8} & 2 \leq x < 3 \\ \frac{7}{8} + \frac{1}{8} = \frac{8}{8} = 1 & 3 \leq x < \infty \\ 0 & x > 1 \end{cases}$	<p>Discrete Random Variables (D.R.V)</p> <p>Probability mass function</p> <p><u>CONDITIONS</u></p> <ol style="list-style-type: none"> <li>(1) <math>f(x_i) \geq 0</math></li> <li>(2) <math>\sum_{i=1}^n f(x_i) = 1</math></li> </ol> <p>Can be in a table</p> <table border="1" style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th>X</th> <th></th> <th></th> </tr> <tr> <th>P(x)</th> <th></th> <th></th> </tr> </thead> </table>	X			P(x)		
X							
P(x)							

## (2) Cumulative Distribution Function F(x)

Continuous	Discrete
$F(x) = \int_{-\infty}^x f(x)dx$ $(1) F(-\infty) = 0, \quad F(\infty) = 1$ $(2) f(x) = \frac{dF(x)}{dx}$ $(3) P(a \leq x \leq b) = F(b) - F(a)$	$F(x) = \sum_{-\infty}^x f(x)$ $(1) F(-\infty) = 0, F(\infty) = 1$ $(2) f(x_i) = F(x_i) - F(x_i - 1)$ $P(a \leq x \leq b) = F(b) - F(a)$

## (3) The Expectation [E(x)] التوقع

اذا تحولت التجربة العشوائيه الى خط الاعداد فان توقع الحد x يمكن حسابه

$E(x) = \mu$	
C.R.V	D.R.V

$$E(x) = \int_{-\infty}^{\infty} x f(x)dx$$

$$E(x) = \sum x f(x)$$

$E(g(x))$	
C.R.V	D.R.V

$$E(g(x)) = \int_{-\infty}^{\infty} g(x) f(x)dx$$

$$E(x) = \sum g(x_i) f(x_i)$$

نحسب  $E(g(x))$  عند كل خانه في ال  $x$  و نضربها في الداله ونجمع

في الاعداد ذات الاحتمالات المتساوية – التوقع هو المتوسط الحسابي  $\mu$

### Properties of Expectation

$$(1) E(a) = a \quad (2) E(ax + by) = aE(x) + bE(y)$$

$$(3) E(ax + b) = aE(x) + b$$

**If  $x$  is C.R.V show that  $E(ax + b) = aE(x) + b$  (هایم)**

$$E(a) = \int_{-\infty}^{\infty} a f(x)dx$$

$$\therefore E(ax + b) = \int_{-\infty}^{\infty} (ax + b) f(x)dx$$

$$= a \int_{-\infty}^{\infty} x f(x)dx + b \int_{-\infty}^{\infty} f(x) dx = aE(x) + b(1)$$

#### التبابن والانحراف المعياري (4) The Variance and standard deviation

هو مقدار البعد او القرب من التوقع لقيم المتغير

$$\sigma = \text{انحراف المعياري} = \sqrt{V(x)}$$

$$V(x) = \sigma^2 = E(x^2) - (E(x))^2$$

$V(x)$	
C.R.V	D.R.V
$V(x) = E((x - \mu)^2)$	$V(x) = E[(x - \mu)^2]$
$V(x) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx$	$V(x) = \sum (x - \mu)^2 f(x)$

#### Properties of Variance

$$(1) V(ax) = |a|^2 V(x) \quad (2) V(ax + by) = a^2 V(x) + b^2 V(y)$$

$$(3) V(ax + b) = a^2 V(x)$$

**if  $x$  is CRV show that  $V(x) = E(x^2) - (E(x))^2$**

$$V(x) = E((x - \mu)^2) \quad \& \quad E(x) = \mu$$

$$V(x) = E(x^2 - 2\mu x + \mu^2) = E(x^2) - 2E(x) * E(x) + E(x)^2 = E(x^2) - (E(x))^2$$

**show that  $V(ax + b) = a^2 V(x)$**

$$V(ax + b) = [E(ax + b)^2 - (E(ax + b))^2]$$

$$\therefore V(ax + b) = [E(ax + b)^2 - (E(ax + b))^2]$$

$$\begin{aligned} &= E(a^2 x^2 + 2abx + b^2) - [a(E(x)) + b]^2 \\ &= a^2 E(x^2) + 2abE(x) + b^2 - [a^2 (E(x))^2 + 2abE(x) + b^2] \\ &= a^2 [E(x^2) - (E(x))^2] = a^2 V(x) \end{aligned}$$

#### العزم الرائي (5) The $r^{th}$ moment

About mean	About origin
$\mu_r$ $\mu_r = E[(x - \mu)^r]$ $\mu_0 = 1$ $\mu_1 = E(x - \mu) = 0$ $\mu_2 = E(x - \mu)^2 = V(x)$	$\mu'_r$ $\mu'_r = E(x^r)$ $\mu'_0 = E(x^0) = E(1) = 1$ $\mu'_1 = E(x) = \mu$ $\mu'_2 = E(x^2)$ $V(x) = \mu'_2 - (\mu'_1)^2$ $\mu_2 = \mu'_2 - (\mu'_1)^2$

## (6) The Moment Generating Function

$M_x(t) = E(e^{xt})$	$M_x(t) = \int_{-\infty}^{\infty} e^{xt} f(x) dx$	$M_x(t) = \sum e^{xt} f(x)$
C.R.V		D.R.V

### Properties of $M_x(t)$

$$(1) \mu'_r = E(x^r) = \frac{d^r M_x(t)}{dt^r} \quad @t = 0$$

$$E(x) = M'_x(0) , E(x^2) = M''_x(0) \dots E(x^n) = M^n_x(0)$$

$$(2) M_{x+a}(t) = e^{at} M_x(t)$$

$$(3) M_{bx}(t) = M_x(bt)$$

(4) generalization

$$M_{bx+a}(t) = e^{at} M_x(bt)$$

**show that  $M_{x+a} = e^{at} M_x(t)$  and  $M_{bx}(t) = M_x(bt)$**

$$M_{x+a}(t) = E\left(e^{(x+a)t}\right) = E\left(e^{at} e^{xt}\right) = e^{at} E(e^{xt})$$

$$M_{bx}(t) = E\left(e^{bxt}\right) = E\left(e^{x(bt)}\right) = M_x(bt)$$

## Probability Distribution

### [1] Discrete probability distribution

An experiment will have two events their probability are **P** and **Q** and test is repeated **n times** so we know the type of distribution

name	n	Dist $f(x)$	$\mu$ or $E(x)$	$V(x)$	$M_x(t)$	Notes
<b>Bernoulli</b>	$n = 1$	$\begin{matrix} p^x q^{1-x} \\ x = 0, 1 \end{matrix}$	$p$	$pq$	$q + pe^t$	عدد مرات التجربة $n$ كل مره يجرب بها التجربة هناك احتمال من اثنين q(fail) او p(success)
<b>Binomial</b> $b(x; n, p)$	$n \leq 50$	$\begin{matrix} C_x^n p^x q^{n-x} \\ x = 0, 1, \dots, n \end{matrix}$	$np$	$npq$	$(q + pe^t)^n$	
<b>Poisson</b> $P(x; \lambda)$	$n > 50$	$\begin{matrix} e^{-\lambda} \lambda^x \\ x! \\ \lambda = np \\ x = 0, 1, \dots, n \end{matrix}$	$\lambda$	$\lambda$	$e^{\lambda(e^t - 1)}$	
						In single trial $p+q = 1$ $x = \text{number pf success}$
<b>-ve Binomial</b> $f(x; l, p)$	Unknown	$\begin{matrix} C_{k-1}^{x-1} p^k q^{x-k} \\ x = k, k+1, \dots \end{matrix}$	$\frac{k}{p}$	$\frac{kq}{p^2}$		$X$ : number of trial until $r^{\text{th}}$ success is observed
<b>Geometric</b> $G(x)$	Unknow	$\begin{matrix} pq^{x-1} \\ x = 1, 2, 3, \dots \end{matrix}$	$\frac{1}{p}$	$\frac{q}{p^2}$	$\frac{pe^t}{1 - qe^t}$	<u>Special case of Geom.</u>  $r=1$ $x$ : no of trial until first success is observed

### Proofs

Find the value of  $E(x), V(x), M_x(t)$  for Bernoulli distribution

X	0	1
$P(X)$	q	p

$$E(x) = \sum x f(x) = (0)(q) + (1)p = p$$

$$V(x) = E(x^2) - (E(x))^2 = (0^2)(q) + (1^2)(p) - p^2 = p(1-p) = pq$$

$$M_x(t) = E(e^{xt}) = \sum e^{xt} f(x_i) = e^{0t}q + e^{1t}p = q + pe^t$$

### Binomial

مفتاح الاختصارات في الحل هو مفهوك ذات الحدين

$$(a + b)^n = \sum_{x=0}^n C_x^n a^x b^{n-x}$$

ممكن اجيبي المعاملات من باسكال

Find the value of  $E(x), V(x)$  and  $M_x(T)$  for Binomial Distribution

$$E(x) = \sum x f(x) = \sum x C_x^n p^x q^{n-x}$$

دائما نحاول تطير  $x$  عشان اللي جوا التجميع يرجع قوس عن طريق فك التوفيق Combination و الاختصار معه

واحاول اعدل التجميع عشان اطلع صوره ذات الحدين اللي فوق دي واقوم اشيل التجميع واحظ القوس

$$E(x) = \sum x \frac{n!}{x! (n-x)!} p^x q^{n-x} = \sum \frac{n!}{(x-1)! (n-x)!} p^x q^{n-x}$$

$$y = x - 1 \rightarrow x = y + 1$$

$$E(x) = \sum \frac{n!}{y! (n-y-1)!} p^{y+1} q^{n-y-1} = p \sum \frac{n(n-1)!}{y! ((n-1)-y)!} p^y q^{(n-1)-y}$$

$$= np \sum \frac{(n-1)!}{y! ((n-1)-y)!} p^y q^{(n-1)-y} = np \sum_0^n C_y^{n-1} p^y q^{(n-1)-y} = np(p+q)^{n-1} = np * 1 = \mathbf{np}$$

$$\begin{aligned} V(x) &= E(x^2) - (E(x))^2 \\ E(x(x-1)) &= E(x^2 - x) = E(x^2) - E(x) \\ \therefore E(x^2) &= E(x(x-1)) + E(x) \rightarrow (1) \end{aligned}$$

$$E(x(x-1)) = \sum x(x-1) \frac{n!}{x! (n-x)!} p^x q^{n-x}$$

$$= \sum \frac{n!}{(x-2)! (n-x)!} p^x q^{n-x}$$

$$\begin{aligned} \text{let } y(x-2) &\rightarrow x = y + 2 \\ &= \sum \frac{n!}{y! ((n-2)-y)!} p^y p^2 q^{(n-2)-y} \end{aligned}$$

$$\begin{aligned} &= n(n-1)p^2 \sum \frac{(n-2)!}{y! ((n-2)-y)!} p^y q^{(n-2)-y} = n(n-1)p^2 \sum C_y^{n-2} p^y q^{(n-2)-y} \\ &= n(n-1)p^2(p+q)^{n-2} \\ \therefore E(x(x-1)) &= n(n-1)p^2 \end{aligned}$$

$$\therefore E(x)^2 = E(x(x-1)) + E(x) = n(n-1)p^2 + np$$

$$\therefore V(x) = E(x^2) - (E(x))^2 = n^2 p^2 - n(n-1)p^2 + np = np(1-p) = \mathbf{npq}$$

$$M_x(t) = \sum e^{xt} f(x) = \sum e^{xt} C_x^n p^x q^{n-x}$$

$$M_x(t) = \sum C_x^n q^{n-x} (pe^t)^x = (\mathbf{p}e^t + \mathbf{q})^n$$

### Poisson

Find the value of  $E(x), V(x)$  and  $M_x(T)$  for Poisson Distribution

$$E(x) = \sum x f(x) = \sum x \frac{\lambda^x}{x!} e^{-\lambda} = e^{-\lambda} \sum \frac{\lambda^x}{(x-1)!}$$

$$\text{let } y = x - 1 \rightarrow x = y + 1$$

$$\therefore E(x) = e^{-\lambda} \sum \frac{\lambda^y * \lambda}{y!} = e^{-\lambda} \lambda \sum \frac{\lambda^y}{y!} = e^{-\lambda} \lambda e^\lambda$$

$$\therefore E(x) = \lambda$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x(x-1)) = E(x^2) - E(x) \Rightarrow E(x^2) = E(x(1-x)) + E(x)$$

$$E(x(X-1)) = \sum x(x-1)f(x) = \sum x(x-1) \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\therefore = e^{-\lambda} \sum \frac{\lambda^x}{(x-2)!} \quad \text{let } y = x-2 \quad , \quad x = y+2$$

$$= e^{-\lambda} \sum \frac{\lambda^y \lambda^2}{y!} = \lambda^2 e^{-\lambda} e^\lambda = \lambda^2$$

$$\therefore E(x^2) = \lambda^2 + \lambda$$

$$\therefore V(x) = \lambda^2 + \lambda - \lambda^2 = \lambda$$

$$\therefore V(x) = \lambda$$

$$M_x(t) = E(e^{xt}) = \sum e^{xt} f(x) = \sum e^{xt} \frac{\lambda^x e^{-\lambda}}{x!}$$

$$= e^{-\lambda} \sum \frac{(e^\lambda \lambda)^x}{x!} = e^{-\lambda} e^{\lambda e^\lambda} = e^{\lambda(e^\lambda - 1)}$$

$$\text{Show that } \lim_{n \rightarrow \infty} b(n, x, p) = p(x, \lambda)$$

Where

$$b(n, x, p) \text{ is P.M.F of binomial distribution}$$

$$p(x, \lambda) \text{ is P.M.F of poisson distribution}$$

$$b(n, x, p) = C_x^n p^x q^{n-x} , p(x, \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

$$\lim_{n \rightarrow \infty} b(n, x, p) = \lim_{n \rightarrow \infty} \frac{n!}{x! (n-x)!} p^x q^{n-x}$$

$$\text{NOTES : } \lambda = np \quad ; \lim_{n \rightarrow \infty} \left(1 + \frac{q}{n}\right)^n = e^q$$

$$p = \frac{\lambda}{n} \quad ; q = 1 - p = 1 - \frac{\lambda}{n}$$

$$\lim_{n \rightarrow \infty} b(n, x, p) = \lim_{n \rightarrow \infty} \frac{n(n-1)(n-2) \dots (n-x-1)(n-x)! \lambda^x}{x! (n-x)!} \frac{\lambda^x}{n^x} \left(1 - \frac{\lambda}{n}\right)^n \left(1 - \frac{\lambda}{n}\right)^{-x}$$

وزیری

$$= \frac{\lambda^x}{x!} \lim_{n \rightarrow \infty} \frac{n}{n} * \frac{n-1}{n} * \frac{n-2}{n} * \dots * \frac{n-x-1}{n} * \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^n * \lim_{n \rightarrow \infty} \left(1 - \frac{\lambda}{n}\right)^{-x}$$

$$= \frac{\lambda^x}{x!} \lim_{n \rightarrow \infty} \left(1 - \frac{1}{n}\right) \left(1 - \frac{2}{n}\right) \dots \left(1 - \frac{x+1}{n}\right) e^{-\lambda} \left(1 - \frac{\lambda}{\infty}\right)^{-x}$$

$$= \frac{\lambda^x}{x!} (1)e^{-\lambda} (1) = \frac{\lambda^x}{x!} e^{-\lambda} = (x, y) \#$$

التجربة تقسم لحدثين  $q$  و  $p$  وتتكرر عدد  $\infty$  من المرات نستخدم التوزيع الهندسي  
احتمال ظهور الحدث الذي احتماله بعد  $x$  من المرات قبلها ظهوره  $q$  من المرات

(Remarks)

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n$$

$$\frac{1}{(1-x)^2} = 0 + 1 + 2x + 3x^2 + \dots = \sum_{n=1}^{\infty} nx^{n-1}$$

$$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + \dots = \sum_{n=0}^{\infty} n x^n$$

$$e^{f(x)} = \sum_{n=0}^{\infty} \frac{(f(x))^n}{n!}$$

$$\frac{d}{dx} \left( \frac{x}{(1-x)^2} \right) = \frac{(1-x)^2(1) + 2x(1-x)}{(1-x)^4} = \frac{1+x}{(x-1)^3}$$

$$\frac{1+x}{(1-x)^3} = 1 + 4x + 9x^2 + \dots = \sum_{n=0}^{\infty} n^2 x^{n-1}$$

Find the value of  $E(x)$ ,  $V(x)$  and  $M_x(t)$  for Geometric Distribution,  $F(x) = pq^{x-1}$ ,  $x = 1, 2, \dots$

$$E(x) = \sum x f(x) = \sum_{x=1}^{\infty} x p q^{x-1} = p \sum_{x=1}^{\infty} x q^{x-1}$$

$$\begin{aligned} E(x) &= p [ 1 + 2q + 3q^2 + 4q^3 + \dots ] \\ &= \frac{p}{q} [ q + 2q^2 + 3q^3 + \dots ] \\ &= \frac{p}{q} \left[ \frac{q}{(1-q)^2} \right] , p+q=1 \rightarrow \frac{p}{q} \left[ \frac{q}{p^2} \right] = \frac{1}{q} \left[ \frac{q}{p} \right] = \frac{1}{p} \end{aligned}$$

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum x^2 f(x) = \sum x^2 p q^{x-1} = p \sum x^2 q^{x-1} = p \left[ \frac{1+q}{(1-q)^3} \right] = p \left[ \frac{1+q}{(p)^3} \right] = \left[ \frac{1+q}{(p)^2} \right]$$

$$\begin{aligned} \therefore V(x) &= \frac{1+q}{p^2} - \frac{1}{p^2} = \frac{1+q-1}{p^2} = \frac{q}{p^2} \\ V(x) &= \frac{q}{p^2} \end{aligned}$$

$$M_{x(t)} = E(e^{xt}) = \sum e^{xt} f(x) = \sum e^{xt} p q^{x-1}$$

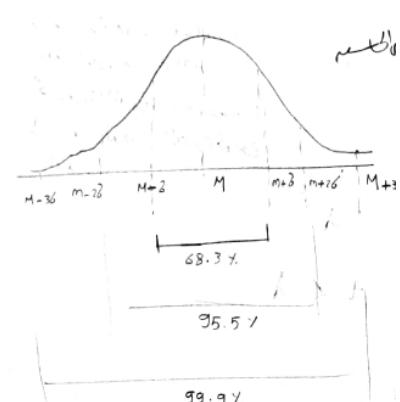
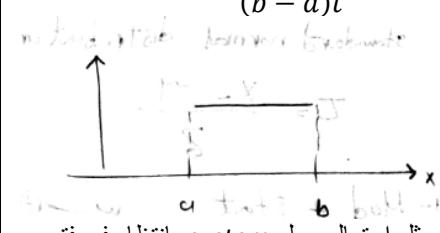
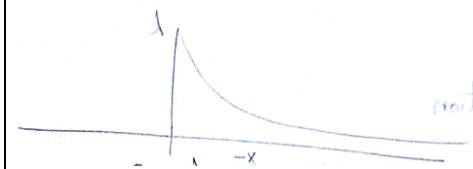
$$= pq^{-1} \sum_{x=1}^{\infty} (e^t q)^x = e^t q + e^{2t} q^2 + \dots$$

نافق لها 1 عشان تبقى شبه الصور

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots = \sum_{n=0}^{\infty} x^n$$

$$= \frac{p}{q} [-1 + 1 + qe^k + (qe^t)^2 + (qe^t)^3 + \dots] = \frac{p}{q} \left[ -1 + \frac{1}{1-qe^t} \right] = \frac{p}{q} \left[ \frac{-1 + qe^t + 1}{1-qe^t} \right] = \frac{pe^t}{1-qe^t}$$

## [2] Continuous probability distribution

Normal Distribution	Uniform Distribution	Exponential Distribution
<p><i>Normal Distribution</i></p> $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$ $E(x) = \mu$ $V(x) = \sigma^2$ $M_x(t) = e^{\mu t + \frac{t^2\sigma^2}{2}}$  <p><i>Standard Normal Distribution</i></p> $\mu = 0, \quad \sigma^2 = 1$ $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}Z^2}$ $Z = \frac{x-\mu}{\sigma}$ <p>عند دراسه الخواص عند تماثل البيانات حول المتوسط</p> <p>نستخدم الاله الحاسبي</p> <p>Mode -&gt; stat-&gt; AC-&gt; shift+1 -&gt; 5-dist -&gt; Q1</p> <p>تحسب من اول الصفر لحد النقطه بتاعتي</p> <p>مجموع من <math>-\infty</math> الى <math>1 = \infty</math> مجموع من <math>0</math> الى <math>0.5 = \infty</math></p>	<p><i>Uniform Distribution</i></p> $f(x) = \begin{cases} \frac{1}{b-a} & ; a < x < b \\ 0 & ; \text{otherwise} \end{cases}$ $E(x) = \mu = \frac{a+b}{2}$ $V(x) = \sigma^2 = \frac{(b-a)^2}{12}$ $M_x(t) = \frac{(e^{bt} - e^{at})}{(b-a)t}$  <p>يمثل احتمال عمل <i>system</i> بانتظام في فترة زمنية محدد او زمن معين و يمثل احتمال وصول سياره او مكالمه تليفونيه ف زمن معين</p>	<p><i>Exponential Distribution</i></p> $f(x) = \begin{cases} \lambda e^{-\lambda x} & ; x > 0 \\ 0 & ; x \leq 0 \end{cases}$ $E(x) = \mu = \frac{1}{\lambda}$ $V(x) = \sigma^2 = \frac{1}{\lambda^2}$ $M_x(t) = \frac{\lambda}{\lambda - t} \quad t < \lambda$  <p>يستخدم في قياس احتمالات اعمار الاجهزه الكهربائيه و تستخدم في حساب الوقت اللازم في تعطل بعض الانظمه الكهربائيه كما تستخدم لحساب الوقت اللازم لوقوع حدث ما</p>

## Normal Distribution

Steps of solving with calculator :

- 1- Convert it to standard  $\tau = \frac{x-\mu}{\sigma}$
- 2- Mode  $\rightarrow$  stat  $\rightarrow$  AC  $\rightarrow$  shift + 1  $\rightarrow$  Distrib  $\rightarrow$  Q(

### Properties of Normal Distribution

1	$P(-\infty < \tau < \infty) = 1$	
2	$P(0 < \tau < \infty) = 0.5$	
3	$P(a \leq \tau \leq b) = Q(b) - Q(a)$ $a, b$ are + ve	
4	$P(-a \leq \tau \leq b) = Q(b) + Q(a)$	
5	$P(\tau \geq a) = 0.5 - Q(a)$	

show that the prob fn for Normal distribution  $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$  is P.D.F

To be PDF We need to prove

$$(1) f(x) \geq 0 \quad (2) \int_{-\infty}^{\infty} f(x)dx = 1$$

to prove

$$(1) f(x) \geq 0 \quad \text{since } e^{-\infty} = 0$$

$$(2) \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$\text{let } y = \frac{x-\mu}{\sigma} \Rightarrow x = \mu + \sigma y \Rightarrow dx = \sigma dy$$

$$I = \frac{\sigma}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dy \rightarrow (*)$$

$$I_1 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dy$$

To Polar

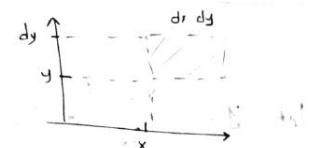
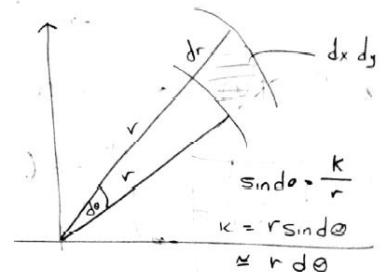
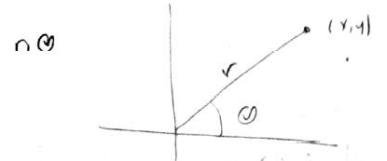
$$I_1^2 = I_1 I_1 = \int_{-\infty}^{\infty} e^{-\frac{1}{2}y^2} dy * \int_{-\infty}^{\infty} e^{-\frac{1}{2}x^2} dx$$

$$I_1 = \iint_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2+y^2)} dx dy$$

$$x = r \cos \theta ; y = r \sin \theta$$

$$r^2 = x^2 + y^2$$

$$dxdy = r dr d\theta$$



$$\therefore I_1^2 = \int_0^{2\pi} \left( \int_0^{\infty} r e^{-\frac{1}{2}r^2} dr \right) d\theta$$

$$= \int_0^{2\pi} \left( \left[ e^{-\frac{1}{2}r^2} \right]_0^{\infty} \right) d\theta$$

$$= - \int_0^{2\pi} [0 - 1] d\theta = [\theta]_0^{2\pi} = 2\pi$$

$$I_1 = \sqrt{2\pi} \quad \text{substitute on (*)}$$

$$I = \frac{1}{\sqrt{2\pi}} * \sqrt{2\pi} = 1$$

Find  $E(x), V(x)$  and  $M_x(t)$  for Normal distribution if  $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$

We find  $M_x(t)$  then we try to get  $E(x)$  and  $V(x)$  from it

$$E(x) = \left[ \frac{d}{dt} M_x(t) \right]_{t=0} ; \quad E(x^2) = \left[ \frac{d^2 M_x(t)}{dt^2} \right]_{t=0}$$

$$M_x(t) = E(e^{xt}) = \int_{-\infty}^{\infty} e^{xt} * \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-\mu}{\sigma})^2}$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{[xt - \frac{1}{2}(\frac{x-\mu}{\sigma})^2]} dx$$

$$xt - \frac{1}{2} \left( \frac{x-\mu}{\sigma} \right)^2 = -\frac{1}{2\sigma^2} [(x-\mu)^2 - 2\sigma^2 xt]$$

$$= -\frac{1}{2\sigma^2} [x^2 - 2\mu x + \mu^2 - 2\sigma^2 tx]$$

$$-\frac{1}{2\sigma^2} [x^2 - 2(\mu + \sigma^2 t)x + \mu^2]$$

نكم المربع

$$\left( \frac{1}{2} \text{معامل الثاني} \right)^2 - \left( \frac{1}{2} \text{معامل الثاني} \right)^2 + \text{باقي جذر الاول}$$

$$= -\frac{1}{2\sigma^2} [(x - (\mu + \sigma^2 t))^2 - (\mu + \sigma^2 t)^2 + \mu^2]$$

$$= -\frac{1}{2\sigma^2} \left[ (x - (\mu + \sigma^2 t))^2 - 2\sigma^2 t \mu - \sigma^4 t^2 \right]$$

نعرض في  $Mx$  و نطلع اللي مفهوش  $x$  برا التكامل

$$M_x(t) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(-2\sigma^2 t \mu - \sigma^4 t^2)} * \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left( \frac{x-(\mu+\sigma^2 t)}{\sigma} \right)^2} dx$$

$$I_1 = \int_{-\infty}^{\infty} e^{-\frac{1}{2} \left( \frac{x-(\mu+\sigma^2 t)}{\sigma} \right)^2} dx$$

$$Z = \frac{x - \mu - \sigma^2 t}{\sigma} \Rightarrow dx = \sigma dz$$

$$I_1 = \sigma \int_{-\infty}^{\infty} e^{-\frac{z^2}{2}} dz$$

$$I_1^2 = I_1 * I_1 = \sigma^2 \int_{-\infty}^{\infty} e^{-\frac{y^2}{2}} dy * \int_{-\infty}^{\infty} e^{-\frac{x^2}{2}} dx$$

$$I_1^2 = \sigma^2 \iint_{-\infty}^{\infty} e^{-\frac{1}{2}(x^2+y^2)} dx dy$$

$$x=r\cos\theta \qquad y=r\sin\theta$$

$$dxdy=rdrd\,\theta$$

$$I_1^2 = (2\pi)(\sigma^2)$$

$$\mathsf{M} \text{ بالتعويض في}$$

$$M_x(t)=e^{\mu t+\frac{1}{2}\sigma^2t^2}$$

$$E(x)=\left[\frac{d}{dt}\;M_x(t)\right]_{t=0}=\left[\left(\mu+\frac{1}{2}\sigma^2(2t)\right)e^{\mu t+\frac{1}{2}\sigma^2t^2}\right]_{t=0}=\mu$$

$$V(x)=E(x^2)-\big(E(x)\big)^2=\sigma^2+\mu^2-\mu^2=\sigma^2$$

$$as\; E(X)=\mu\; \; and\; E(x^2)=\left[\frac{d^2}{dt^2}\;M_x(t)\right]_{t=0}=\sigma^2+\mu^2$$